MTH 408/522: Numerical analysis

Homework III: Aitken's, Steffensen's, Horner's and Müller's methods

(Due 16/09/19)

Problems for turning in

1. Prove Theorems 1.8.1 and 1.9.1 of the Lesson Plan.

2. A sequence $\{p_n\}$ is said to be *superlinearly convergent* to p if

$$\lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|} = 0.$$

   (a) Show that if $p_n \to p$ of order $\alpha > 1$, then $\{p_n\}$ is superlinearly convergent.

   (b) Show that if $p_n \to p$ superlinearly, then

$$\lim_{n\to\infty} \frac{|p_{n+1} - p_n|}{|p_n - p|} = 1.$$

3. Let $P_n(x)$ be the n^{th} Taylor polynomial for $f(x) = e^x$ expanded about $x_0 = 0$.

   (a) For a fixed x, show that $p_n = P_n(x)$ satisfies the hypothesis of Theorem 1.8.1.

   (b) Does Aitken's method accelerate convergence in this situation?

Problems for practice

1. In each of the following, generate the first five terms of the sequence $\{\tilde{p}_n\}$ obtained using the Aitken's Δ^2 method.

   (a) $p_0 = 0.75$, $p_n = \sqrt{e^{p_{n-1}}/3}$, $n \geq 1$.

   (b) $p_0 = 0.5$, $p_n = (2 - e^{p_{n-1}} + p_{n-1}^2)/3$, $n \geq 1$.

2. In each of the following, use Steffensen's method to approximate solutions to the following equations to within 10^{-5} accuracy.

   (a) $x^3 - 2x - 5 = 0$.

   (b) $3x^2 - e^x = 0$.

3. Find approximations to within 10^{-4} to all real zeros of the following polynomials using Müller's method and then reducing to polynomials of lower degree to determine any complex zeros.

   (a) $f(x) = x^4 - 4x^2 - 3x + 5$.

   (b) $f(x) = x^4 - 2x^3 - 12x^2 + 16x - 40$.

   (c) $f(x) = x^3 - 7x^2 + 14x - 6$.

   (d) $f(x) = x^5 - 11x^4 - 21x^3 - 10x^2 - 21x - 5$.